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$$w \frac{d^2 v}{dx^2} + (2 \frac{dw}{dx} + Pw) \frac{dv}{dx} + (\frac{d^2 w}{dx^2} + P \frac{dw}{dx} + Qw)v = R \dots \dots \dots (3),$$

with the conditional equations :

$$\frac{d^2 w}{dx^2} + P \frac{dw}{dx} + Qw = 0 \dots \dots \dots (4),$$

$$\frac{d^2 v}{dx^2} + [(2/w) \frac{dw}{dx} + P] dv/dx = R/w \dots \dots \dots (5).$$

w being supposed known from (4) gives

$$w^2 \frac{dv}{dx} e^{-\int P dx} = A + \int w R e^{\int P dx} dx \dots \dots \dots (6),$$

$$\text{and } v = B + A \int \frac{dx}{w^2} e^{-\int P dx} + \int \frac{dx}{w^2} e^{-\int P dx} \int w R e^{\int P dx} dx \dots \dots \dots (7).$$

Now (4) is of the same form as (1) excepting that the right member is 0 ; so that if we have a solution of (4) we have that of (1) when $R=0$.

The given equation is

$$\frac{d^2 y}{dx^2} - \frac{2x}{1+x^2} \frac{dy}{dx} + \frac{2}{1+x^2} y = 0 \dots \dots \dots (8);$$

then $P = -2x/(1+x^2)$, and a particular solution is

$$y = x \dots \dots \dots (9), \text{ or } w = x \dots \dots \dots (10).$$

$$\text{Then (7) gives } v = B - A \int \left(\frac{dx}{x^2} + 1 \right) = B + A(x - [x/1]),$$

and $y = vx = Bx + A(x^2 - 1)$, the required solution.

[As will be seen from the last solution both forms are correct. The first form is given as the answer, on page 336 of *Byerly's Integral Calculus*. EDITOR.]

Also solved by O. W. ANTHONY, W. C. M. BLACK, J. SCHEFFER, G. B. M. ZERE and P. S. BERG.

58. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics in Columbian University, Washington, D. C.

A line passes through a fixed point and rotates uniformly about this point. Another line passes through a point which moves uniformly along the arc of a given curve and rotates uniformly about this point. Develop a method for finding the locus of intersection of these two lines. Apply to case of circle and straight line.

Solution by the PROPOSER.

Let $y=f(x)$ be the equation of the curve, taking the fixed point as origin. Let (x_1, y_1) be the position of the moving point at time t . Call θ_1 the angle which the line through the fixed point originally makes with axis of x ; also, let ω_1 be the rate of angular rotation. Then the equation of line is

$$y=\tan(\omega_1 t + \theta_1)x;$$

$$\text{whence } t=(\omega_1/1)[\tan^{-1}(y/x)-\theta_1] \dots\dots\dots(1).$$

Also the equation of other line is

$$y-y_1=\tan(\omega_2 t + \theta_2)(x-x_1);$$

$$\text{whence } t=(\omega_2/1)(\tan^{-1}\frac{y-y_1}{x-x_1}-\theta_2) \dots\dots\dots(2).$$

$$ds_1^2=dx_1^2+dy_1^2. \quad ds_1/dt=v_1. \quad \therefore v_1^2=[1+f'(x_1)^2] \frac{dx_1^2}{dt^2}.$$

$$t=\frac{1}{v_1} \int [1+f'(x_1)^2]^{\frac{1}{2}} dx \dots\dots\dots(3). \quad y_1=f(x_1) \dots\dots\dots(4).$$

To solve the problem, then, we integrate (3), solve the resulting equation for x_1 , substitute this value in (4), and then substitute the values of x_1 and y_1 in (2), after which t is to be eliminated between the resulting equation and (1). To apply this method to the case of the straight line

$$y_1=f(x_1)=0. \quad x_1=v_1 t.$$

$$\therefore \text{From (2), } t=\frac{1}{\omega_1} \tan^{-1}\left(\frac{y}{x-v_1 t}\right)-\theta_1.$$

$$\text{Then } \frac{1}{\omega_1} \left(\tan^{-1} \frac{y}{x} - \theta_1 \right) = \frac{1}{\omega_1} \left(\tan^{-1} \frac{y}{x-(v_1/\omega_1)(\tan^{-1}[y/x]-\theta_1)} - \theta_2 \right).$$

$$\text{Let } y=\rho \sin \phi; \quad x=\rho \cos \phi.$$

$$\text{Then } \frac{1}{\omega_1}(\phi-\theta_1) = \frac{1}{\omega_2} \left[\tan^{-1} \left(\frac{\rho \sin \phi}{\rho \cos \phi - (v_1/\omega_1)(\phi-\theta_1)} \right) - \theta_2 \right].$$

$$\therefore \frac{\rho \sin \phi}{\rho \cos \phi - (v_1/\omega_1)(\phi-\theta_1)} = \tan[\theta_2 + \frac{\omega_2}{\omega_1}(\phi-\theta_1)].$$

$$\therefore \rho = \frac{(v_1/\omega_1)(\phi-\theta_1) \tan[\theta_2 + (\omega_2/\omega_1)(\phi-\theta_1)]}{\cos \phi \tan[\theta_2 + (\omega_2/\omega_1)(\phi-\theta_1)] - \sin \phi},$$

the polar equation of the curve.

The case of the circle leads to complicated results. The case of two fixed points is interesting. The equations of the intersecting straight lines may be written

$$y = \tan(\omega_1 t + \theta_1)x \dots\dots\dots(1), \text{ and } y = \tan(\omega_2 t + \theta_2)(x - a) \dots\dots\dots(2).$$

$$\text{From (1) } \tan\phi = \tan(\omega_1 t + \theta_1).$$

$$\therefore \phi = \omega_1 t + \theta_1. \quad t = (1/\omega_1)(\phi - \theta_1).$$

$$\therefore \rho \sin\phi = \tan(\omega_2 t + \theta_2)(\rho \cos\phi - a);$$

whence $\rho = [a \tan(\omega_2 t + \theta_2)] / [\cos\phi \tan(\omega_2 t + \theta_2) - \sin\phi]$, or

$$\rho = \{a \tan [\frac{\omega_2}{\omega_1}(\phi - \theta_1) + \theta_2]\} / \{\cos\phi \tan[\frac{\omega_2}{\omega_1}(\phi - \theta_1) + \theta_2] - \sin\phi\},$$

$$= \{a \sin [\frac{\omega_2}{\omega_1}(\phi - \theta_1) + \theta_2]\} / \{\sin[(\frac{\omega_2}{\omega_1} - 1)\phi + \theta_2 - \frac{\omega_2}{\omega_1}\theta_1]\} \dots\dots\dots(m).$$

If they both start from the initial horizontal position at the same time,

$$\theta_1 = \theta_2 = 0, \text{ and } \rho = a \frac{\sin[(\omega_2/\omega_1)\phi]}{\sin\{[(\omega_2/\omega_1) - 1]\phi\}}.$$

A large number of curves is included in equation (m).

This problem was also solved by C. W. M. Black. His solution will appear in the next issue.

MECHANICS.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

41. Proposed by O. W. ANTHONY, M. Sc., Professor of Mathematics, Columbian University, Washington, D. C.

If the earth were a perfect sphere and had a frictionless surface, what would be the motion of a particle placed at a given latitude?

Solution by the PROPOSER.

Adopt as coördinates the latitude of the particle and the distance measured